THESIS

THE STRENGTH OF CURVED STRUTS,

FOR THE DEGREE OF

B. S.,

SCHOOL OF ARCHITECTURE,

BY

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Strength of Curved Struts.

It often becomes necessary, in ornamentative construction, to use timbers of curved outline, which are to be subjected to compressive stresses.
The timber attains its form, not by any deflection of its fibres, but by being sawn out of straight pieces to the required curvature.
Thus the fibres, which in the first case would be practically of equal lengths, under this treatment are of various lengths, between the limits of zero, (for a point at middle ordinate of curve,) and the length of the chord of the curvature of the strut.

This inequality of fibre-length of course materially weakens the strength of the timber, and it is in the investigation and comparison of the strength of curved struts with that of straight ones; that this thesis is to deal.

To obtain data for the discussion of this question, I decided to make experimental tests upon small pine struts of varying curvature.

The standard strut used was eighteen inches, vertical length. The section was a square, one and one half inches on a side.
All struts sawn from same plank, a clear white pine, slightly damp from exposure, in shed.

The middle ordinate of curvature of the centre line of the strut, or, what I shall hereafter designate the deflection of the strut, varied by one-quarter inches, from zero, (for a straight strut) to two inches as a maximum.

I carefully worked out three good specimens for each deflection, making thirty in all. All tests of specimens were made with the Riehlé Testing Machine at the University shop. The tests being as follows.

Average ultimate strength being reduced for square inch.

÷ Straight struts — or struts of zero deflection.

No 1. failed at	10950
No 2.	10250
No 3.	11400
Average ultimate strength being	4829 lbs.

All three of the specimens failed in the middle, by crushing only.

Struts of 1/4" Deflection.

Number One . . . . 7000 pounds
" " " Two . . . . 7000 " "
" " " Three . . . . 7000 " " "
Average ultimate strength being 31.11 pounds

No 1. failed by crushing at middle and at three inches low each side of middle. It also split on the convex side one-third the distance from the edge.

No 2. failed by crushing near the middle, and split similarly to No. 1.

No 3. - duplicate of No 1.

Struts of 1/2" Deflection.

No 1.	5840 lbs
No 2.	5800 " "
No 3.	5150 " "

Average ultimate strength being 2288 pounds.

No 1. crushed at centre and split in two places, one-third from each side of strut.
No 2 split one third from convex edge and crushed at centre.
No 3. split one third from concave edge and crushed at centre.

Struts of 3/4" Deflection

No 1.	4400 Pounds
No 2.	4000 " "
No 3.	3890 " " "

Average ultimate strength being 1840 pounds
No 1 crushed in three places and split in centre
No 2. crushed in two places, split on convex edge.
No 3 crushed in four places, splitting in centre.

Struts of 1" Deflection

No 1. . . . . 4200 pounds
No 2. . . . . 3300 " " "
No 3 . . . . 2800 " " "

Average ultimate strength being 1527 pounds.

No 1. crushed at centre only
No 2. crushed in three places and split near edge.
No 3. crushed at centre, only

Struts of 1¼" Deflection

No 1 . . . . 2395 pounds.
No. 2 . . . . 3060 " " "
No. 3 . . . . 2925 " " "

Average ultimate strength = 1330 pounds.

Nos. 1, 2, & 3 all failed by crushing at centre.
No. 1 and 3 split near upper end, and sheared off one corner.

Struts of $1\frac{1}{2}$" Deflections

No 1. . . . . 2700 pounds.
No 2. . . . . 2300 " "
No 3. . . . . 2100 " "

Average ultimate strength = 1162 pounds

All three specimens broke by crushing near centre.

Struts of $1\frac{3}{4}$" Deflection.

No 1. . . . . 2305 pounds
No 2. . . . . 2000 " "
No 3. . . . . 2610 " "

Average ultimate strength = 1025 pounds

No 1. broke at centre and split on convex edge.
No. 2. failed on each side of middle and split through center.
No. 3 crushed at centre only.

Struts of 2" Deflection

No 1. . . . . 2200 Pounds
No 2 . . . . 2000 " " "
No 3 . . . . 2000 Pounds.

Average ultimate strength = 918 Pounds.
All the specimens split entirely in two about one third the width from the convex side.

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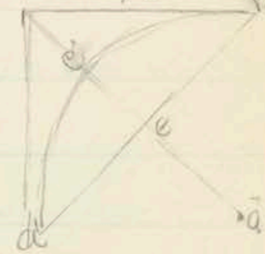
To investigate these results I platted the curve shown in Plate I.
The units for ordinates represent seventy pounds, the ordinates of points on the curve corresponding to the breaking strength of the timber for those points. The abscissas express the ratio between the deflection and length of any strut, five of the abscissas' units forming one per cent of length. (as I shall hereafter call the said ratio.

Thus, by the Plate, any white pine strut 18 inches long, one and one half inches square, with a given deflection may be investigated.
Example:— A strut of above material, length and section, has a deflection of $\frac{5}{8}$ of an inch. What is its ultimate strength?
The percent of length equals $\frac{5}{8} \div 18 = .347$ or 3.47%
$3.47 \times 5 =$ no. of abscissa units to be measured. $= 17.35$, approximately $17\frac{1}{3}$. (shown on curve as +)
We find the point on the curve whose abscissa is $17\frac{1}{3}$ and derive its ordinate from the vertical scale, taking 70 pounds for every unit.
The result is the breaking strength of the strut, in this case $70 \times 28.5 = 1995 =$ ultimate strength.
And so for any deflection between zero and two inches, accurate results may be obtained.
Owing to lack of further experiments, could

not continue the true curve, but produced the tangent to the curve from the last true point. Hence with deflection greater than _two_ inches results would only be approximate by use of this plate.

Thus, a strut eighteen inches long, with deflection of $3\frac{3}{5}$ inches has a _per cent of length_ $= 3\frac{3}{5} \div 18 = 20\,\%$.
The point on the produced tangent, whose abscissa expresses this _per cent of length_, gives us an ordinate of _four_ units;—whence $4 \times 70 = 280$ pounds = approximate breaking strength of given strut.
The practical limit of deflection for _any_ strut is 20%, as may be seen from figure A.

A



The arc _a.b.c_ is the centre line of the strut.
It will never occur greater than a quadrant.
Then line _ab_ $= \sqrt{2} = .141^{+}$. _be_ $= R - \frac{1}{2}\,ab = 1 - .707$
Maximum ordinate hence equals .293 of radius,

or. $293 \div .141^{+} = .207 = 20\%^{+}$ of the length of the strut.
However for completeness and purposes of investigation I have produced tangent to horizontal axis, or what is the same thing have measured back the requisite number of per cent units (four) upon an auxiliary horizontal axis passing through lower extremity of tangent.

<u>Erratum</u>, Each figure on auxiliary axis should read one less, as the intercept of tangent on axis of x is 24, not 25.

-:- Practical Adaption of the Curve for all Struts -:-

Though platted from results of tests made on pieces of standard length, section and material this curve can be adapted to the investigation of the strength on any strut, irrespective of size or material, providing its deflection gives a per cent of length between zero and twenty four.

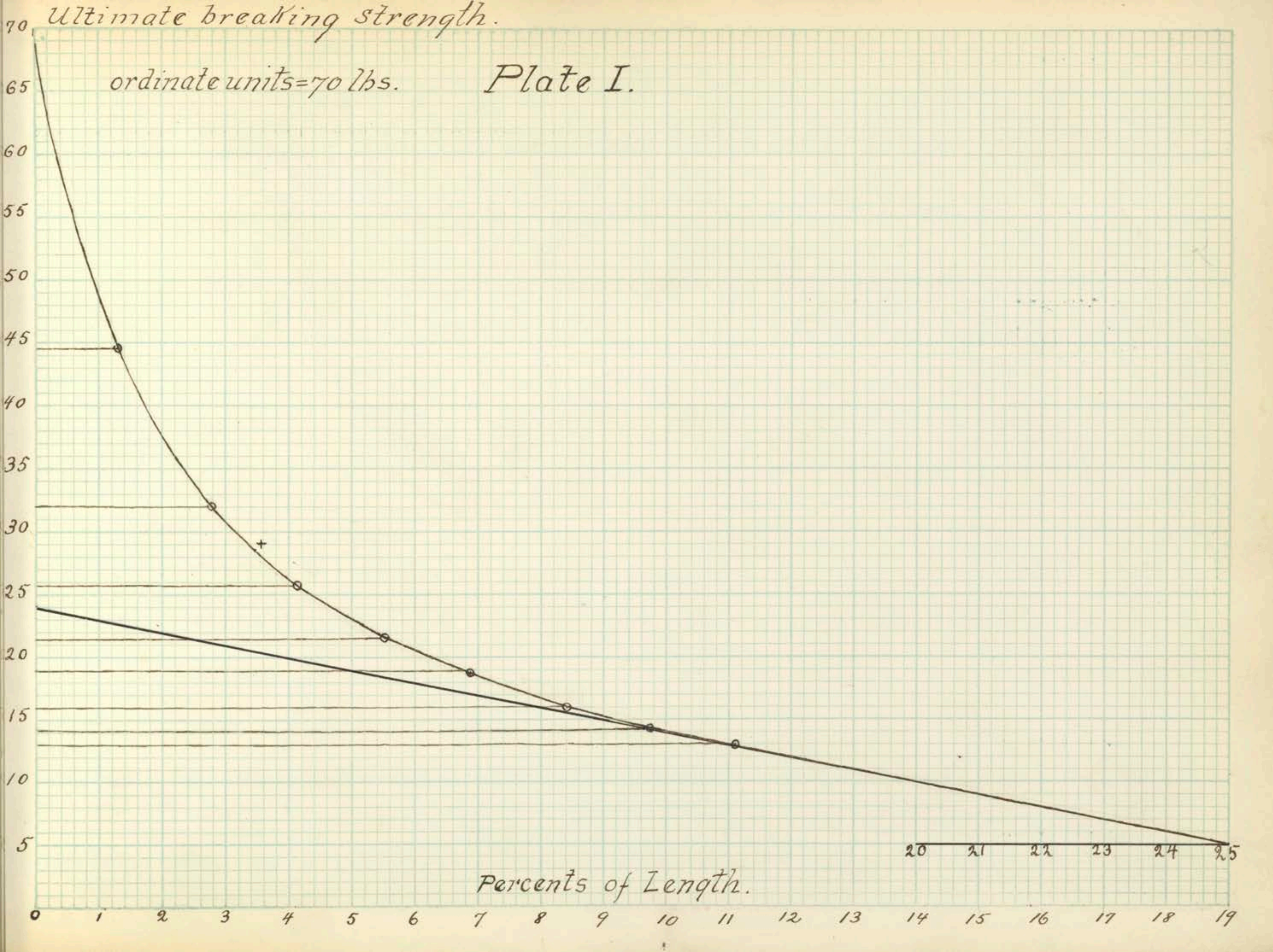
To accomplish this I duplicated the curve in Plate II, and divided its maximum ordinate (the ordinate representing the breaking strength of a straight strut) into one-hundred equal parts.
Each unit will then represent one per cent of the strength of the straight standard strut; and consequently the number of units expressed in the ordinate of any point of the curve, will be the per cent of strength of the curved strut for that point, as compared with the straight one.
It is easily seen that now the ordinates and abscissae being expressed relatively, all restrictions as to length, size, shape, curvature and material disappears, and we have a curve of practical value in investigating the strength of _any_ strut. For the curve gives us the ratio of the strength of a curved strut with that of a straight one,

whence the absolute strength of the curved strut is readily obtained by multiplying this ratio by the ultimate strength of a straight strut of same length and section.

Ultimate strength should be determined by formula $w = C \div 1 + .00396\frac{l^2}{d^2}$. in which C is coefficient for different materials. l = length of column in inches. Note, By length of curved column we mean the length of the chord of its axis.) d is the side in inches. w = ultimate load in pounds per square inch. C is coefficient for crushing, expressed in pounds per square inch.

If we call a the per cent of strength of any curved strut, then its ultimate strength per square inch, will be expressed —,

$$w' = aw.$$



Ultimate breaking strength.
ordinate units=70 lbs.
Plate I.
70
65
60
55
50
45
40
35
30
25
20
15
10
5
20 21 22 23 24 25
Percents of Length.
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

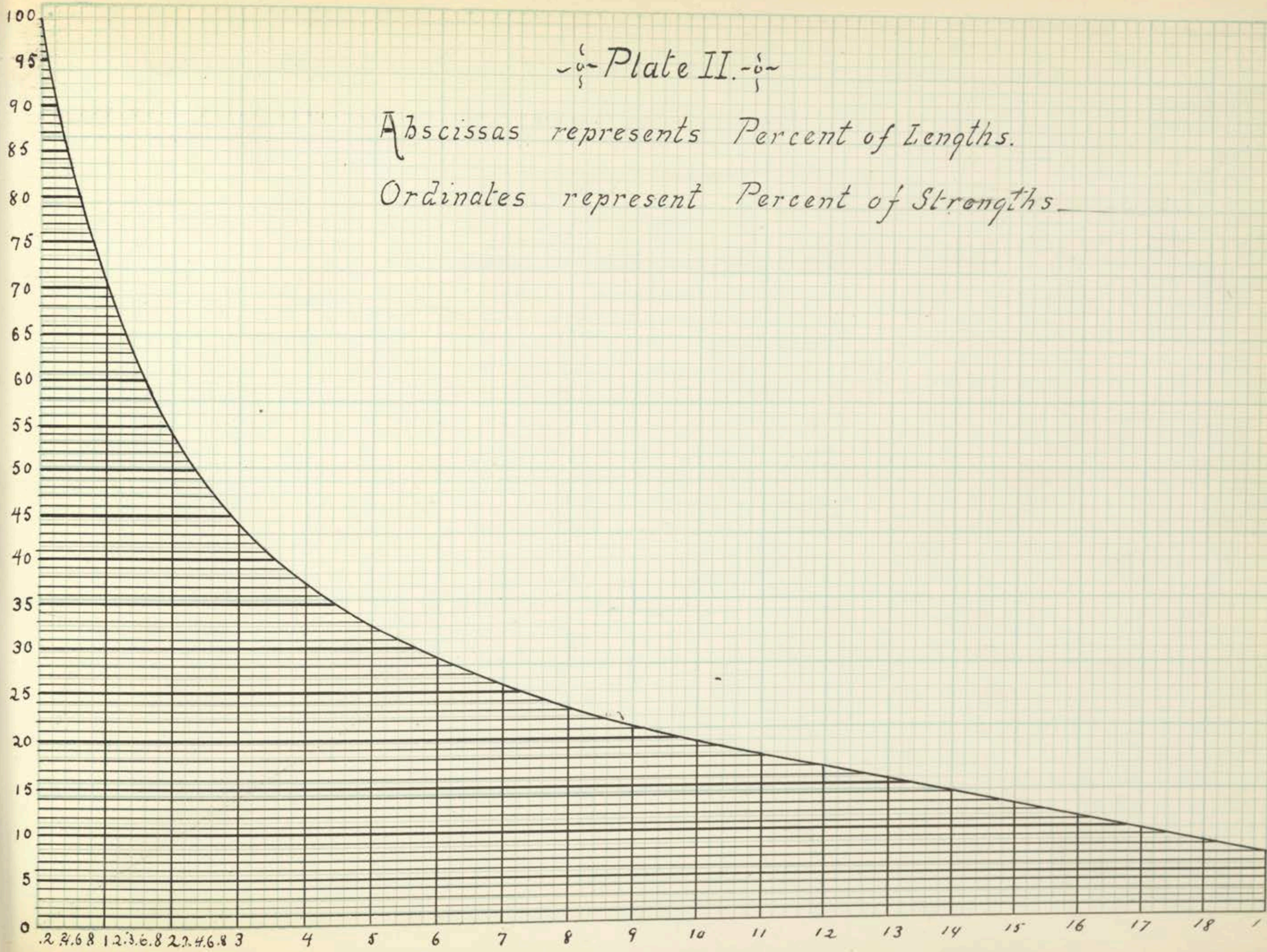


Plate II.
Abscissas represents Percent of Lengths.
Ordinates represent Percent of Strengths
100
95
90
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
10
5
0
.2 .4 .6 .8 1 .2 .3 .6 .8 2 .2 .4 .6 .8 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

Plate III.

Table of Percents.

Percent of Length.		Percent of Strength
1		70.6
2		54.0
3		44.3
4		37.5
5		32.5
6		29.0
7		26.6
8		23.2
9		21.2
10		19.5
11		18.0
12		17.0
13		16.5
14		14.0
15		12.5
16		11.3
17		9.5
18		8.3
19		7.0
20		—